Students' Proof Ability: Exploratory Studies of Abstract Algebra Course

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Abstract
This research aims to describe students' ability in proof construction, proof understanding, and critical thinking as result of implementation of Guided Discovery Learning trough the Motivation to Reasoning and Proving Tasks (GDL-MRP Tasks) in Abstract Algebra course. This study used explanatory sequential method, that is quantitative methods at the first phase and qualitative at the second. The first phase of research used pre-experimental design with two static group comparisons. The populations of this study were 205 students participating in five classes of Abstract Algebra course. Two classes are randomly selected, one class as the experimental group who received GDL-MRP Tasks and the other class as a control group who received Direct Teaching (DT). Initial tests conducted in both classes for determine the initial ability of students. They are classified into three levels of initial ability (low, medium, and high). The analysis of posttest concluded that the greatest influence of implementation GDL-MRP Tasks occur at students in low-to-medium initial ability category. The second phase of research concluded that the ability of the student's proof construction can be determined by six categories: initial step, flow of proving, related concepts, arguments, key expressions, and language of proof.
Keywords: Critical thinking, Discovery learning, Proof construction

1. Introduction

Mathematics is the science that uses axiomatic deductive reasoning, doesn't accept the truth just based on inductive events. Generalizations are just based on mere examples, contrary to deductive reasoning in mathematics. Based on the axiomatic deductive, the mathematics learning can't be separated from learning of proof. Hanna (2010) notes that write a mathematical proof will help students' understanding of the mathematical object. Reid in Cyr (2013) notes that trains students to write a mathematical proof, will develops their deductive reasoning.

Previous studies have shown that construct and understand the proof is a difficult activity. The study of Indonesian Mathematics and Science Teacher Education Project (IMSTEP) in 1999 concluded that the mathematical activity which difficult by students to learn and teachers to teach is justification or proof (Suryadi, 2012). Cyr (2013) notes that proof writing is an activity that most complex and difficult for students. This was caused by the difficulty students in understanding the basic structure of deductive reasoning. When students write proof, they often have errors in sequencing argument.

The ability to formulate a good proof can't be separated from its ability to critical thinking. Critical thinking is a basic capability that should be owned by everyone to be implemented in everyday life. Emphasis on improving the critical thinking skills is part of mathematics learning. Axiomatic deductive reasoning in mathematics makes the material highly instrumental in training and improved critical thinking skills.

Information or opinion of a person is not necessarily a truth that can be trusted. It’s necessary critical attitude with rational thinking to analyze the information. Ennis (1996) stated that critical thinking is a thought process that aim to make rational decisions related to what is believed. Because the decision making will take place continuously in life, then critical thinking is an important asset for everyone in everyday life.

Beaumont (2010) stated that in order to improve critical thinking skills in learners, required the provision of training in the form of tasks that require a high reasoning in solving. The task of observing, assumptions identifying, a material challenge to understand, interpret assignment or interpretation of a phenomenon, characterized by discovery and investigation tasks, tasks to analyze and evaluate, and the task to make a decision, is believed to develop critical thinking skills.

The main aim of the Abstract Algebra course is develop a critical thinking and mathematical proof students' ability. The strategies that support these objectives need to be given serious consideration in order to achieve the course objectives optimally. Learning model that emphasizes the active involvement of students is discovery learning. Alfieri (2011) notes that the effects of unassisted-discovery tasks seem limited, whereas enhanced-discovery tasks requiring learners to be actively engaged and constructive seem optimal. The discovery learning with guidance can increase the activity of learners and the construction of knowledge becomes optimal.

Alfieri (2011) suggest a process of discovery should be equipped with one of the task guided by scaffolding on certain parts, the tasks that ask students to explain their ideas and ensure that they are accurate idea by providing feedback in a timely manner, or the tasks contains examples and directives that guide the work of the task completion well. Based on the results of that study, it can be concluded that Alfieri (2011) recommended a guided discovery learning more than discovery learning only. In the guided discovery learning, it is necessary to determine the accuracy lecturer guidance that directs students to invent new knowledge, without too much direct involvement with students. For that, we need a model of guidance by providing work direction using a task that is packaged in the student worksheet.

Takac (2009) developed a special task called the Motivation to Reasoning and Proving Tasks (MRP Tasks). MRP Tasks are (1) tasks that looks to have an easy solution, but after dealing with problem exhaustively, it has a different perhaps surprising solution, (2) tasks that can be solved intuitively, but students are not sure of solution‘s correctness, or (3) task that has several possible solutions and students have to decide (and verify), which one is correct. In his research, Takac was concerned with influence of MRP tasks on students’ willingness to reasoning and proving. The MRP tasks type of assignment can help students to increase motivation to learn and realize the importance of proof in mathematics. It can develop students' intrinsic motivation to realize that proof is very important in mathematics. The MRP Tasks can develop students' critical thinking. Students become aware that they need to verify the views, statements or information other people before being accepted. Critical thinking is important not only in mathematics but also in everyday life. Based on the above description, implementation GDL-MRP Tasks are expected can develop students’ proof and critical thinking abilities in Abstract Algebra course.

2. Method

This study uses a combination of quantitative-qualitative research. The scheme of research can be described in the Figure 1. The first step of the study used a model of randomized group posttest only design involving samples of two classes, namely the experimental class and control class (Millan, 1996). The populations of this study were 202 students who participating in the Abstract Algebra course that disaggregated into 5 classes. Two classes are randomly selected from population, one class as the experimental group who received GDL-MRP Tasks and another class as a control group who received DT. Sample classes are not formed by randomly placing research subjects into the sample classes, but using existing classes.

At the end of the lesson, the students in both classes received the final test, which measures the ability of the construction of proof, understanding of proof and mathematics critical thinking. To look more deeply of the effect of learning strategies on aspects of research, this study involved initial ability of student with 3 levels, namely high, medium, and low. The leveling is based on the results of initial tests.

Analysis of the data is aims measured the students' ability in three aspects namely proof construction, proof understanding and mathematics critical thinking ability. Data have been classified by learning model and initial ability levels of students. Hypothesis test is done for each pair of students in the same initial ability students. Before doing the hypothesis test, test of normality of distribution and homogeneity of data variance were firstly conducted. The test of normality of distribution data used the Shapiro-Wills and homogeneity of data variance used Levene's test. The final data analysis aims to determine the effect of interaction between the model of learning and the initial ability of students. To determine the effect of these interactions, researcher used two-way ANOVA. Before doing the two-way ANOVA test, test of normality of distribution and homogeneity of data variance were firstly conducted.

The second step research aims to describe the quality of students' proof construction. This study used a grounded theory method through the steps of open coding, selective coding and theoretical coding (Jones and Irit, 2011). Subject of research are 41 students who got GDL-MRP tasks. The preliminary analysis conducted on four proof construction ability matters that tested on the first phase of this study.

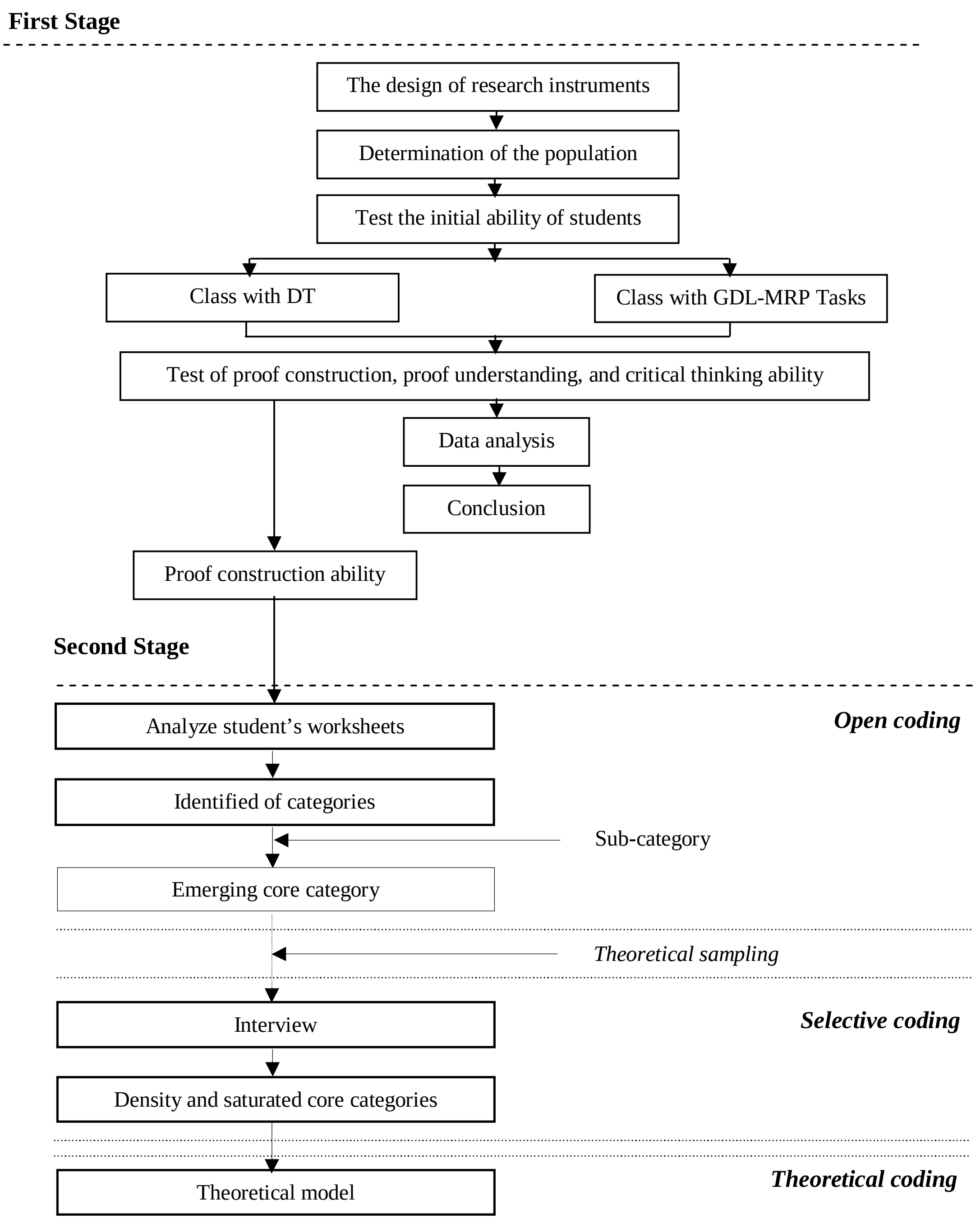

Figure 1. Research scheme

3. Results

The influence of learning factor in both classes were analyzed by based on the results of the final test about proof construction, understanding proof and mathematics critical thinking ability. Final test consists of 9 items developed based on the lattice in accordance with aspects of the observations in the study, and had been tried in other students to get a good instrument. The recapitulation of posttest in the range of 0 to 100 score are listed in Table 1.

Table 1. The recapitulation of posttest

Class	IAS	N	Aspect					
			Proof construction		Proof understanding		Critical thinking	
			Mean	SD	Mean	SD	Mean	SD
E	L	6	70.83	17.84	86.67	18.07	48.00	12.74
	M	29	55.90	23.33	77.41	21.66	56.38	17.04
	H	6	79.00	22.25	94.17	3.76	55.00	10.08
C	L	8	18.88	9.51	51.88	27.25	31.38	24.05
	M	26	41.35	26.11	68.08	19.96	37.08	21.66
	H	6	69.83	29.62	77.50	14.05	46.67	25.02

Description: E = Experiment Class, C = Control class, IAS = Initial ability of students**,** N = number of students, SD = Standard Deviation

The results of influence test of learning factor to proof construction ability with α = 0.05 significance level are shown in Table 2**.**

Table 2. Test summary of t-test of proof construction ability

Category		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Differ-ence	Std. Error Differ-ence	95% Confidence Interval of the Difference Lower	Upper
Low	Equal variances assumed	3.869	0.073	7.067	12	0.000	51.958	7.352	35.940	67.976
	Equal variances not assumed			6.478	7.127	0.000	51.958	8.021	33.061	70.856
Medium	Equal variances assumed	0.213	0.646	2.183	53	0.034	14.550	6.667	1.179	27.922
	Equal variances not assumed			2.169	50.498	0.035	14.550	6.708	1.080	28.021
High	Equal variances assumed	0.531	0.483	.606	10	0.558	9.167	15.125	-24.534	42.867
	Equal variances not assumed			0.606	9.281	0.559	9.167	15.125	-24.891	43.224

Based on the results of statistical tests in Table 2, it can be concluded that (1) the proof construction ability of the student at low category in the experimental class better than students at the same category in the control class, (2) the proof construction ability of the student in medium category in the experimental class better than students in the same category in the control class, while (3) the proof construction ability of the student at high category in the experimental class did not differ significantly compared to students in the control class.

The results of influence test of learning factor to proof understanding ability with α = 0.05 are shown in Table 3.

Tabel 3. Test summary of t-test of proof understanding ability

Category		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2 tailed)	Mean Differ-ence	Std. Error Differ-ence	95% Confidence Interval of the Difference Lower	Upper
Low	Equal variances assumed	2.451	0.143	2.700	12	0.019	34.792	12.885	6.719	62.865
	Equal variances not assumed			2.867	11.892	0.014	34.792	12.134	8.326	61.257
Medium	Equal variances assumed	0.109	0.743	1.656	53	0.104	9.337	5.637	-1.969	20.643
	Equal variances not assumed			1.664	52.954	0.102	9.337	5.611	-1.918	20.592
High	Equal variances assumed	38.043	0.000	2.806	10	0.019	16.667	5.940	3.433	29.901
	Equal variances not assumed			2.806	5.714	0.033	16.667	5.940	1.955	31.379

Based on the results of statistical tests in Table 3 it can be concluded that (1) the ability of proof understanding for students at low category in the experimental class better than students in the same category in control class, (2) the ability of proof understanding for student at medium category in the experimental class better than students in the same categoriy in control class, and (3) the ability of proof understanding for student at high category in the experimental class is better than the students in the same category in control class.

The results of influence test of learning factor to critical thinking ability with $\alpha = 0.05$ are shown in Table 4.

Tabel 4. Test summary of t-test of mathematics critical thinking ability

Category		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Differ-ence	Std. Error Differ-ence	95% Confidence Interval of the Difference Lower	Upper
Low	Equal variances assumed	4.126	0.065	1.530	12	0.152	16.625	10.868	-7.055	40.305
	Equal variances not assumed			1.668	11.054	0.123	16.625	9.967	-5.300	38.550
Medium	Equal variances assumed	1.059	0.308	3.692	53	0.001	19.302	5.228	8.817	29.788
	Equal variances not assumed			3.644	47.413	0.001	19.302	5.297	8.650	29.955
High	Equal variances assumed	0.923	0.359	0.757	10	0.467	8.333	11.011	-16.201	32.868
	Equal variances not assumed			0.757	6.582	0.475	8.333	11.011	-18.043	34.710

Based on the results of statistical tests in Table 4 it can be concluded that (1) the ability of students to critical thinking at low category in the experimental class did not differ significantly compared to students in the same category in control class, (2) the ability of students to critical thinking at medium category in experimental class is better than students in the same category in control class, and (3) the ability of students to critical thinking at high category in the experimental class did not differ significantly compared to students in the same category in control class.

Terms of normality of the data values for proof construction ability of students in the experimental class and control class is not satisfy, so the two-way ANOVA test can't be used. To describe the relationship between the learning factors and initial ability to proof construction, we analyze the graph of interaction that presented by Figure 2.

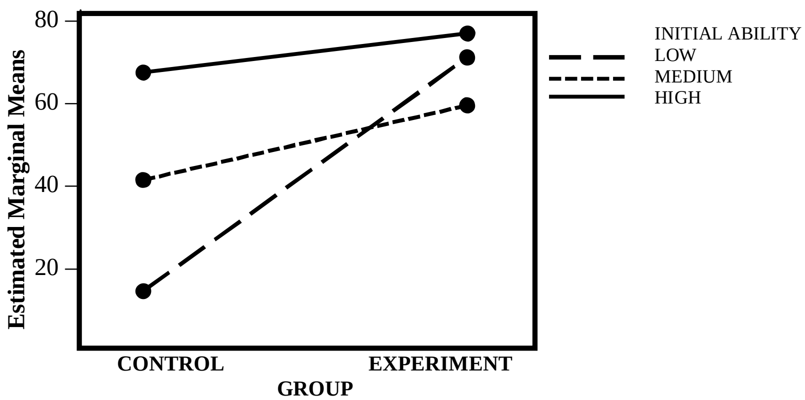

Figure 2. Interaction between learning model and initial ability toward students' proof construction

Based on Figure 2 it can be concluded that; (1) the learning and initial student ability factors together provide a positive influence on the ability of the proof construction, and (2) the greatest effect of the interaction of learning and initial ability factors on the proof construction occurred in the low category, medium category at the second, and the smallest at high category.

Terms of normality of the data values for proof comprehension ability of students in the experimental class and control class is not filled, so the two-way ANOVA can't be used. The analysis was performed descriptively to graph of interaction between factors of learning and initial student ability of proof understanding shown in Figure 3.

Based on Figure 3 it can be concluded that; that (1) the learning and the initial student ability factors are together provide a positive influence on the ability of proof understanding, and (2) the greatest effect of the interaction of learning and initial ability factors on proof understanding occurs in the low category, the second order in high category and the smallest in the medium category.

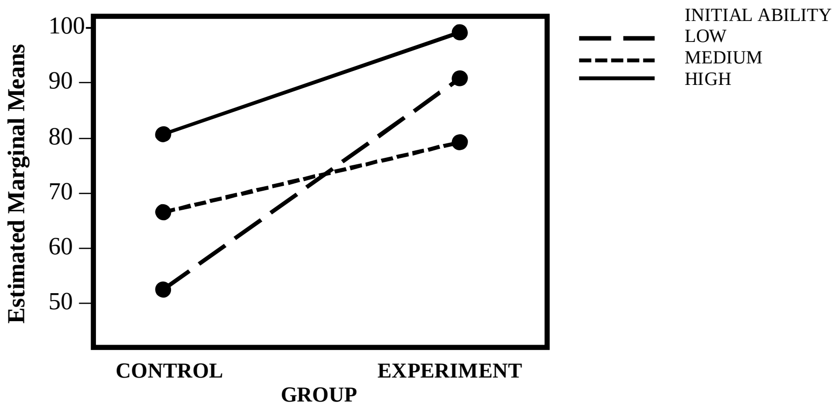

Figure 3. Interaction between learning model and initial ability toward students' proof understanding

Terms of normality of the data value for the critical thinking skills of students in the experimental class and control class is not satisfied, so the two-way ANOVA test can't be used. The analysis was performed descriptively to interaction graph between factors of learning and the initial ability of the students' critical thinking skills shown by Figure 4 below.

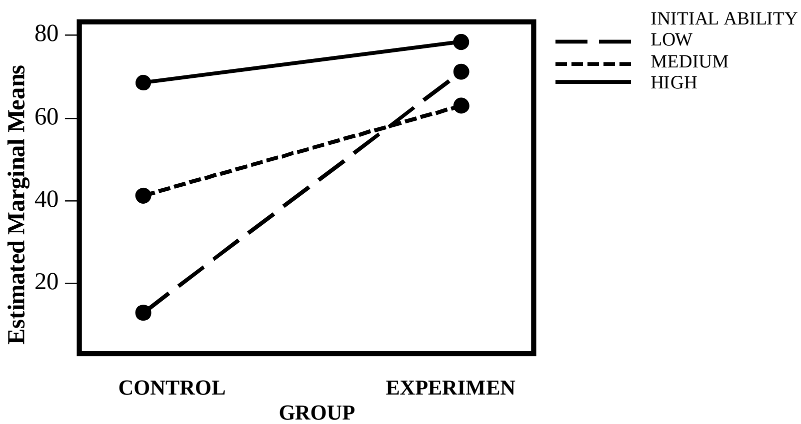

Figure 4. Interaction between learning model and initial ability toward students' critical thinking

Based on Figure 4 it can be concluded that; (1) the learning and student ability factors are together provide a positive influence on the ability of mathematics critical thinking, and (2) the greatest effect of the interaction of learning and initial ability factors on the ability of proof understanding occurs in medium category, the second order in low category and the smallest in the high category.

4. Discussion

Research findings showed that the average value of proof construction ability of students in the experimental class higher than students in the control classes. Achievement is valid for all categories of initial ability students, although based on statistical tests, students with low and medium categories that differ significantly. These results indicate that the GDL-MRP Tasks have a good influence on proof construction students' ability in Abstract Algebra course.

The statistical test showed that the ability of proof construction in the experimental class students who are at a low level category is significantly better than the students who are at the same level in the control class. This is supported by the findings of the interaction effect test about influence learning factors to initial ability students who show that the learning factor is very effective in low and medium categories. These findings suggest that GDL-MRP Tasks is suitable for improve the proof construction ability.

Should be assumed that the use of worksheets as guided in the discovery process, have a major role in strengthening the student to be able to construct a good proof. Worksheet a guide provided by the lecturer, helping students to more easily understand the concept. This was revealed in an interview with one of the students from the experimental class, related her opinion about the learning process is carried out. Interviewer asked respondent to compare GDL-MRP Tasks in Abstract Algebra course with other learning. Piece of the interview is as follows:

Subject	Content
…	…
I	When be compared with [learning] that we use non-conventional in Abstract Algebra course, which is according to R [interviewer called respondent 's name] in terms of understanding the concept or material, it is considered easier?
R31	I think that we better use.
I	Yes. Any comments?
R31	Because in my opinion, the students there are required to find a concept with guidelines. So students do not get bored. If we just listen to the lecturer talked, then there are students asked questions, it could be either sleepy or bored. If that's the discussion all of students are required to think.
I	Yes ... sleepy ... the constraints of conventional learning yeah ... That's the contention R [interviewer called respondent's name] yes ... non-conventional learning that we were used more interesting. When I asked yesterday related discussion yes ... discussion that occurred in the R31 group [interviewer called respondent's name] like what?
R31	Exchanging ideas. No idle. Suppose we find it difficult matter and hold one of our more understand it, explain to other friend. Or suppose the three of them do not understand, we ask the lecturer.
…	

Based on the piece of the interview, it can be concluded that the R31 felt the guide (worksheet) are discussed together with her friends, is helpful in understanding the concept. Students assume that the learning in her class better than direct instruction.

Proof understanding is the ability of student to digest proving flow that studied. The ability of students is measured through accuracy in implementing proving flow into specific examples or problems, according to the flow of proving. Reasoning in the verification process, can be described precisely in the appropriate meaning. The results of statistical test showed that the students' ability of the proof understanding in the experimental class better than students in the control class. This result is valid for all levels of initial ability categories. This suggests that GDL-MRP Tasks is very effective in the experimental class to improve students' proof understanding ability.

A students' worksheet was arranged gradually from the introduction of the concepts, examples and non-examples, properties that arise, proving of properties, proof understanding, challenge to the test of mathematics critical thinking, have a significant role in enhancing students' understanding of the flow of proving. Learning about proving flow requires guidance by lecturers because of the structure of proof in mathematics is specific, which requires precision and step by step guarantees that each step is selected, has a logical mathematical relationship.

Implementation of guided discovery learning is helps students to understand the flow of valid proof. This is in accordance with Selden and Selden (2013) that learning is not enough simply to understand verification through homework or tests, without the assistance of teachers (lecturers). Selden and Selden (2013) recommend the learning through the presentation of a number of statements and the students are asked to prove a statement. Guided discovery learning provides wider opportunities for students to understand the proof. Ask and answer activity in their discussion requires students to each other arguments, giving valuable experience to enhance the ability of proof understanding.

In their discussion is required to explain and reinforce their opinions to others. A person who has understood the groove of proof in a mathematical expression, should be able to explain to others that the flow is valid proof. This is in accordance with Lee and Smith (2002) that should be able to convince the truth mathematics statement to others. Lee and Smith (2002) argues that the sociological perspective, the proof must be understood by those involved in the study.

The practice of proof involves three distinct processes that are closely interrelated evidence searches, organizing proof, proof and explanation for others. Explaining the proof to others not merely presenting evidence. Mathematician must earnestly strive to convince others that the proof presented is really valid and can be understood by the communities involved.

Most of the tasks contained in students' worksheet are provides a guide to understanding the flow of students to practice proof given. Students are required to be able to implement flow statement into corresponding example. Implementation a flow of proving into a groove suitable examples, improving students' understanding of 'storyline' logical proof, so avoid just 'able to memorize' mathematical proof.

The results of statistical test showed that the critical thinking skills of students in the experimental class better than students with the same category in the control class. Although the results of statistical tests for students in low and high category did not differ significantly, the mean score for both categories are in the experimental class higher than the control class. The results of the analysis of the interaction effect between learning factors and the initial ability of students confirm that findings. Learning factors exert influence on all categories and the greatest influence on the students in medium category.

One of the factors that can be suspected as a cause of higher critical thinking skills in the experiment class than in the control class is the use of teaching materials in the form of a well-developed worksheet by MRP Tasks. Using worksheet containing tasks characterized by MRP in any learning, enabling the growth of a critical attitude to the students. It supported research results Takac (2009) that the use of MRP Tasks makes the formation of a critical stance on learner related mathematical statement.

In the experiment class, researchers have prepared a special teaching materials developed by worksheet with discovery aspect of the MRP Tasks. Before being used in the experimental class, the teaching materials had tested in the feasibility class, part of the population, one week before the lecture in the experimental class. This provides an opportunity for researcher (lecturer) to make improvements to the deficiencies that occur in the development of teaching materials.

The availability of good teaching materials and designed exclusively by lecturer to bring up the situation in the classroom didactic, very important. This is in accordance with Suryadi (2013) which states that the process of learning mathematics can essentially be viewed as a process of forming a new mental objects based on the association between mental objects that have been previously owned. The process is triggered by the availability of teaching materials that design by lecturer to result a didactic situation that allows students.

The worksheet guide the process of the invention which is for students coupled with limited involvement of lecturer in the group discussions and classical, is an act of scaffolding that help students achieve optimal learning outcomes. Interactivity was assisted by student centered worksheet, and limited assistance from lecture, provide an opportunity for each student to actively engage in the learning process and remains consistent with the purpose of learning. This is in accordance with Suryadi (2013) which states that the complexity of the didactic situation, a challenge for lecturer to be able to create a situation so that the appropriate pedagogical growing interactivity is capable of supporting the achievement of the potential ability of each student.

The using worksheet with MRP Tasks in each course is opportunity for lecturer to create a didactic situation for enhancing of students critical thinking. The tasks are done by discussion, to train students to be able to convey an argument or arguments clashing with their friends. Discussions and defending an argument is an appropriate vehicle in critical thinking training. The research findings through interviews indicate that disagreements between students sometimes appear in their discussion.

To describe of students' proof construction ability, in the second step of research, we analyze the student work sheets about proof construction. The focus of the analysis are:

(1). Was written the initial step leading to the proper construction of proof?
(2). What proving strategy is chosen?
(3). How are the students understood of the assumptions or things that are known in the matter?
(4). How are the students prepared or written arguments? Are the arguments presented valid and meaningful in understanding the range of classroom community?
(5). How are the students thinking in the overall job? Is illustrated with coherent reasoning or logic leap there?
(6). How many key expressions that appears in the proof?
(7). Were notation, mathematical terms and symbols used appropriately?
(8). How are the students mastering related concepts that needed in constructing the proof?
(9). Whether the language of proof is meaningful in community outreach class?

Based on that analysis, are founded the variability of student works. The rate of mistake for four proof constructions matters are shown in Table 5.

Table 5. Percentage of mistakes in proof construction

No	Focus	Mistake (%)			
		1	2	3	4
1	Initial step	29.27	48.78	21.95	63.41
2	Strategy of proving	39.02	39.02	29.27	68.29
3	Understanding of the assumptions	34.15	46.34	41.46	75.61
4	Written arguments	43.90	58.54	48.78	68.29
5	Students thinking	43.90	53.66	43.90	75.61
6	Key expression	41.46	43.90	48.78	68.29
7	Using notation, mathematical terms and symbols	34.15	58.54	11.20	41.46
8	Students mastering of related concepts	43.90	58.54	48.78	73.17
9	Using language of proof	31.71	51.22	41.46	75.61

The finding mistake with their work in the open coding phase indicates that the variability of student work can be reviewed by 9 focus of the analysis. In a further analysis of the selective coding stage of 9 focus of the analysis can be presented as a 6 core categories are shown in Figure 5.

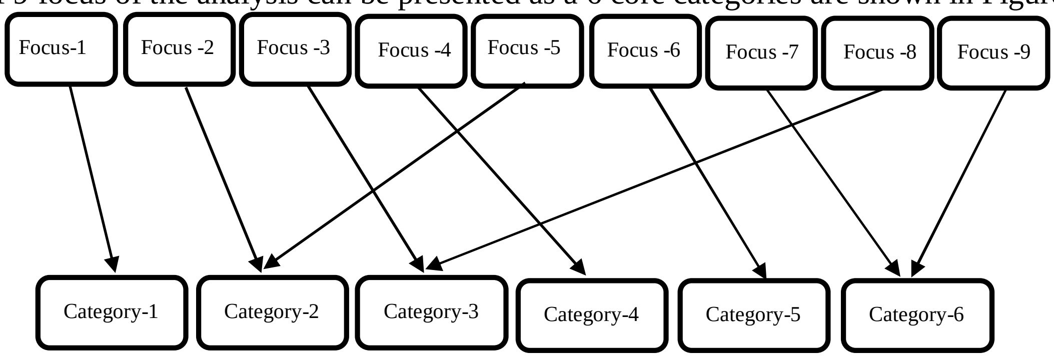

Figure 5. Scheme of formation categories

Description:
Category - 1 : The initial step
Category - 2 : Flow of proof
Category - 3 : Related concepts
Category - 4 : Arguments
Category - 5 : Expression key
Category - 6 : Language of proof

Based on the test results of proof construction ability, students are sorted into three levels of ability namely high, medium and low. Interviews conducted for all six categories studied by researchers to 6 student respondents (2 from the low, 2 from the medium, 2 from the high). The purpose of the interview is to get a description of the proof construction abilities by sixth categories. Based on students work analysis and interviews with researchers had findings that the ability of the proof construction can be rated into three ability levels (high, medium, low).

Students with a high level of proof construction has the ability to identify the assumptions and known in the statement to be proved, and is able to properly utilize it as capital in determining the initial step proof construction. Students are able to describe the assumptions and known to the operational form in the first step of proof. The first step had described that thinking has led to a student who requested proof. In most cases, from the first steps disclosed indicate that students already know the final destination of the requested proof. They have a proficiency in using the strategy of proof is clear. Proof construction had reflected a coherent flow of thought, according to the proof strategy that used. The steps are performed groove reflect coherent thought and does not contain a leap of logic.

Students with high levels ability of proof construction shows a good understanding of all related concepts needed in the construction of the proof as a whole. Concepts are put to good use in determining the proof process steps. The students can elaboration of the meaning concept and operational support in the flow of proving. They have skills in preparing exact argument of the steps taken in the proof. The final conclusion or pre-conclusions of overall proof construction is based on the argument that right.

Key expressions are subject matters that should appear in a valid proof. Students with a high level of proof construction ability were able to bring all key expressions appropriately arranged in a proof structure. They are able to present proof construction with proof language in a communicative and meaningful community in outreach class. The words, phrases and sentences expressed in a simple and clear. Proof construction is supported by the use of notations, symbols and mathematical terms appropriately.

Students with medium levels of proof construction ability have weaknesses to identify some of the assumptions and known in the statement to be proved. As a result, students experienced a mistake in determining the initial step. Students experienced a mistake in describing the assumptions or what is known in a statement (matter) into operational form in the early steps of proof. From the initial step proof construction is not reflected in the right direction. They were able to describe the chosen strategy in the construction of the evidence compiled, but the consistency of use these strategies are not properly maintained. As a result, the majority of proof seems to think that a coherent flow, but on the other part occurs leap of logic.

Students with the medium level ability understand the construction of proof most relevant concepts required in the construction of the proof, but most of the other concepts that need not controlled. Proof construction inaccuracies characterized by a student in the use of related concepts. They reveal proof containing arguments with low accuracy rate, albeit not occur in the entire proof. The final or pre-conclusions based on the decided without proper arguments.

Students with medium level ability proof construction were only able to bring some of the key expressions that should appear in the proof. The proof construction with efforts to bring key expression, but not stated properly. They use the language of proof, either in the form of a sentence or a mathematical expression that does not have a clear meaning in a range of classroom community. Notations, symbols or mathematical terms are not used appropriately.

Students who are on a low level proof construction ability was not able to identify assumptions, or what is known in a statement (matter) which will be proved that experience errors or inability to determine the initial step. They are experiencing a mistake or incompetence in choosing the strategy used to construct the proof. The proof contained the reasoning that is not coherent and is characterized by the presence of a leap of logic.

Students with low level construction ability are not mastered evidence related concepts needed in the construction of evidence. Students do not understand the relationship between the concepts of supporting proof. They were unable to express argument on the assertion that should require a supporting argument. Conclusions are not based on right argument. Students with low level proof construction abilities can't bring up the key expression in the proof. They can't reveal proof of the communicative construction. The words, phrases or expressions used does not meaning or cause ambiguity. The unclear language tinged with proof of mistake in most of the writing of notations, symbols or mathematical terms in the proof.

5. Conclusion

GDL-MRP Tasks have a positive influence on students' ability to construct the proof, understand the proof and critical thinking. The results showed that students with low and medium category of initial ability are gaining influence higher than students with high category. Mentoring by highly capable students has a positive effect on the ability of students with low or medium category. Initial ability of student is valuable information in the implementation of the course. Therefore, the mapping of the ability of students needs to be done at the beginning of the course, before the entire material studied. Information on initial ability can be used to determine the groups of student in discussion learning.

The results of a grounded theory study showed that quality of proof construction can be determine by six categories namely the initial step, the flow of proof, related concepts, arguments, key expressions, and language of proof. To be able to construct a good proof, six categories need to be considered. Therefore, in Abstract Algebra lectures should be emphasized on student awareness of the importance of the six categories in constructing the proof.

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