# CLUSTER ANALYSIS OF CLASS FOUR STUDENTS BASED ON SCHOOLS PROBLEM MATH STORY MATHEMATICS

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**Abstract**. Many elementary students still find it difficult to solve story problems, because to solve them is not just using a formula, but students must also understand the problems in the story problem and can use appropriate strategies to solve them. Problems regarding the number of students who have difficulty in solving story problems indicate an error in the teaching and learning process so that there needs to be an improvement. But before making improvements, the teacher must first analyze any mistakes experienced by students in working on story problems. By knowing these errors, teachers are expected to make appropriate improvements for the next teaching-learning process. One procedure that can be used in analyzing student errors in working on story problems is the Newman procedure. Based on these problems, the purpose of this study is 1) Identifying the mistakes made by grade IV students of Elementary School Serang City in solving fraction story problems, 2) Knowing the causes of grade IV students making mistakes in working on fraction story problems.

The method of this research is content analysis. The analysis is carried out on students' error sheets in working on mathematical story problems and the results of student interviews about the causes of these errors as well as teacher interviews to get solutions that can reduce these errors.

The results of this study: 1) Mistakes made by 4th grade students in working on fraction story problems were errors reading as much as 5.81%, errors understanding problems 24.41%, transformation errors as much as 17.83%, errors of ability the process as much as 24.03%, and conclusions writing errors as much as 27.91%, 2) The causes of student errors in working on fraction story problems there are 4 factors namely because of difficulty in reading, difficulty in understanding the problem, not understanding the concept of fraction counting operations, and student inaccuracy, 3) Solutions that can be done to reduce students' mistakes in working on fraction story problems are: the teacher should direct students to apply problem-solving steps to students, can introduce various strategies that can be used to solve story problems. the teacher should use a learning model that can improve students' ability to solve math story problems.

Keywords: analysis, story matter, fraction.

#### 1. Introduction

Mathematics is a deductive, formal, hierarchical science and uses symbolic language which has a dense meaning. Mathematics is called deductive science because mathematics does not accept generalizations based on observation, experimentation, and experimentation (inductive). Mathematics is very useful in everyday life because humans are not free from problems related to mathematics, such as the process of buying and selling and measurement. Mathematics becomes a tool also in learning other sciences, such as in physics, economics, history and so on. Because of the many benefits of mathematics for humans, mathematics subjects are studied at all levels of education including in elementary schools.

Primary school students who study mathematics are expected to form a systematic mathematical mindset, logical, critical with great care. In reality, in elementary school, there are still many students who think that mathematics is one of the subjects that is difficult to learn and understand so that there are still many students who have difficulty learning mathematics. Difficulty in learning mathematics is a situation in learning that is characterized by certain obstacles in achieving student mathematics learning outcomes. According to Ramirez, Gunderson, Levine & Beilock (2013), mathematics learning difficulties faced by students are marked in some common mistakes in working on math problems, namely mistakes in understanding symbols, place values, calculations, incorrect use of processes, and writing that cannot be read. Mathematics given is taught in elementary schools including arithmetic, algebra, geometry, and statistics as well as opportunities. The fraction is one of the material in arithmetic that is taught in elementary school in addition to chopping numbers and integers. Fractions are numbers that students can use in daily life. Fractions can help students in the process of buying and selling, measuring, and other student activities and can be used by students when learning other sciences, for example in studying natural sciences, social sciences, arts, and others. Because of the many benefits of this fraction, students should be able to understand and master fraction material. Students' mastery in fractional material can be a prerequisite for learning the next mathematical material such as the measurement of the circumference, area, and volume. But if students do not understand fraction material, then it is likely students will have difficulty in learning the material.

Fraction material provided in class 4 includes material simplifying fractions, fraction addition and subtraction operations, and fraction story problems. According to Ashlock (Ashlock, D., & Kim, E. Y. (2016)explains that story problems can be presented in oral or written form, written story questions in the form of sentences that illustrate activities in daily life. Story questions given to students can be taken from daily life and student experience so that it can make students understand the purpose of the problem. This shows that the question story is a question that requires understanding when students solve it. To solve a story problem, students need to understand the problem, understand what is known and what is asked, can choose a strategy or approach to solving the story problem, can solve the story problem and can check the results of the answer. Many elementary students still find it difficult to solve story problems, because to solve them is not just using a formula, but students must also understand the problems in the story problem and can use appropriate strategies to solve them.

Based on preliminary observations at Elementary School in Serang, Indonesia, the mathematics learning outcomes of some grade IV students are still low. According to the class teacher, students still have difficulty in solving math story problems. Because of the number of questions given, questions in the form of stories that still have many mistakes. Apart from initial observations in class, researchers observe students longer and always ask the teacher or friend in solving story problems.

When solving story problems in the form of problem-solving questions, the teacher does not provide students with stages of problem-solving. This kind of learning makes it difficult for students to develop mathematical abilities. Therefore the teacher's role is very influential in developing students' abilities. Problems regarding the number of students who have difficulty in solving story problems indicate an error in the teaching and learning process so there needs to be an improvement. But before making improvements, the teacher must first analyze any mistakes experienced by students in working on story problems. By knowing errors experienced by students, teachers are expected to make appropriate improvements to the learning process

# 2. Method

The method of this research is content analysis, namely research conducted by studying texts, documents, or books to draw conclusions based on the context of their use. (Krippendorff, 2009, 2018), (Downe-Wambold, 1992), (Lombard, Snyder-Duch, & Bracken, 2004), (McMillan, 2000) and (Stemler, 200)In this study, what will be analyzed are documents in the form of student error sheets in working on mathematical story problems and the results of student interviews about the causes of these errors. The subjects used to conduct this research were grade IV students at Elementary School in Serang with 23 students. The instrument in this study is the researcher himself, but after the focus of the study becomes clear, it is possible to develop a simple research instrument, which is expected to complement the data and compare it with data that has been found through observation and interviews. So, in addition to being an instrument, researchers also need to make other instruments to assist researchers in collecting data needed in research. The instrument is in the form of a test in the form of mathematical story questions relating to fractional material and interview guidelines.

## 3. Results and Discussion

There were 23 students present at the time. Tests were given as many as 5 story questions in the form of a description of the broken material. The test is carried out in the morning from 8:00 to 9:00 or for 60 minutes. After the test is completed, the students' answers are examined and examined and grouped, namely the group of student answers that are all correct and the group of student answers that still have errors. There are 9 correct groups of answers and 15 errors that still have answers. Furthermore, the answer groups of students who still have errors are analyzed based on the Newman method, namely analyzing: reading errors, understanding problems, transformation, arithmetic and writing error answers. Interviews were conducted with students to find out more about the causes of student errors in answering questions about fractions. Interviews were carried out after the examination of test results and carried out by six students and researchers. One student interviewed two students, while the researcher conducted three students.

The findings of the data in this study are as follows:

1. The results of the analysis of students' answers aim to find out the errors of students in working on fraction story problems. The data is described based on the type of error in the form of a description that is put together in a table based on the type of error made on each item.

a. Reading error

The following are the descriptions made by researchers regarding the mistakes made by students in working on fraction story problems.

Based on the results of the interview it was found that the reading errors in this study were carried out by three students. The following are examples of the results of interviews conducted with students:

Q: Read about number one

S: A ... ni me ... m..be ... li (read spelled)

P: Try reading about number two

S: You can't, ma'am

Results of interviews with inclusive students:

Q: Read about number one

S: E ... no, ma'am

P: Come on mom help to read it

S: Ani ... (imitating the words of the researcher then silence while

bowed)

The following interviews with other students:

Q: Can you read about number one?

S: (keep quiet)

P: Come on, read slowly

S: A .... here .... remember ... be ... li (read spelled)

P: Now read about number two

S: Ra ... ni .... me..mi..li ... ki (read spelled)

P: Yes, I will practice reading a lot later,

S: Yes ma'am

Based on the results of the interview, it was found that there was a student who could not read (inclusion student) and two students who read spelled. There were 15 reading errors.

b. Misunderstanding

1) Do not write down what is known and asked

On the subject 1,2,3,4,5,6,7, This type of error is quite a lot of mistakes by students in this study.

Subject 1

S1 does not write what is known and asked so S1 can not be a known understanding of the problems that exist in the story problem. This error was made by students in questions no. 1,2,3,4 and 5. Following one of the clippings the answer sheet S1 made that mistake.



Figure 1. Error S1 does not write the thing that is known and thing asked

S2 made the same mistake in questions no. 1 to no. 5. This shows the lack of understanding of students to distinguish things that are known and things that are asked, or do not understand the contents of the problem story. Next clipping the answer sheet S2 made the error.



Figure 2. Error S2 that does not write a thing known and asked.

In question no. 1, S9 does not write the unit of kg on the unknown, while on the question, the subject does not write the question word so that the sentence is incomplete namely: "mango and orange" should be "how many fruits did Rani buy?". The sentences made by S9 are incomplete so students will have difficulty when drawing conclusions. The following cutouts of S9 answer sheets make these mistakes.



Figure 3. Error S9 in writing thingsunknown thing

Subject 10

In addition to subject 9, subject 10 is also incomplete writing things known as in question no. 2, the subject only writes a third tape and Rani is eighth.

Error writing things that are asked

Subject 10

Subject 10 has written things that are known and things that are asked, but in terms of questions S10 still writes incomplete sentences in problem 1 and S10 is incomplete so that if it is read it has no meaning and is

difficult to understand and is not the thing that is asked about the problem. Next is the cutout of the answer sheet that shows the error.



Figure 4. Error S 10 writes what is being asked.

c. Transformation Error

1) Error in determining arithmetic operations

Subject 1

In the answer to question no. 2, S1 writes a division operation that requires Operation because the question asked is "how many meters does Rani's tape get?" Next is the cutout of answer sheet S1 for the error.



Figure 5. Error S1 in determining the count operation.

Based on these errors, it is certain that S1 will make mistakes at a later stage. Subject 2

S2 made a mistake in determining the count operation. In this case, there are two arithmetic operations. S2 has made the second operation, but in the first arithmetic operation performed the sum is made into operation, while the second arithmetic operation has actually been made as an aid.

The following image cutouts S2 errors that indicate these errors.



Figure 6. S2 error in establishing a count operation

Subject 3

S3 made a mistake in determining the arithmetic operations on question no 3. S3 used the required division operation using judgment. In question no. 5, S3 Operations add and add operations. The desire in the first part uses Operations and the second part uses operations as well.

Subject 4

S4 made a mistake in determining the operation at count 4, S4 made preparations and completion. Should manage the addition and Support operations.

Subject 5

S5 made a mistake in determining the arithmetic operations on questions no. 2 and 3. In question no. 2, S5 used the arithmetic operation that required division. While in question no. 3, S5 also uses the count calculation operation required for the Registration Operation. The following is the error answer sheet made by S5.



# Figure 7. S5 error in determining the count operation Subject 6

S6 made a mistake in the arithmetic operation in question no. 4 only explained one arithmetic operation, which was added, which received two arithmetic operations, namely addition and turning. In question no.5, S6 is wrong in issuing arithmetic operations because it uses addition and division operations, which only require operations in the first part and in the second part. Subject 7

S7 made a mistake in performing the count operation in question no. 5 because it uses both the count operation and the addition and recovery. The first part should use the count operation and the second part also uses the count operation Support.

Subject 8

S8 made a mistake on question no 3. S8 only used one Application count operation. Supposed to be surgery. In question no 5. S8 made a mistake also using the Calculate Addition and Summon Operations. The operation should have been using the Money Count Operations and the second operation was also using the Money Count Operations.

Subject 9

S9 made a mistake with question number 2. S9 used a division operation. The operation should have been approved. In question no. 3, S9 uses the division operation as well, replacing it using the operation with Update. In question no. 4, S9 uses only one addition calculation operation. Supposedly to solve this problem must use the addition and calculation operations. In problem no. 5, S9 uses the sum calculation operation. It should use a count operation.

Subject 10

In problem number 1, S10 has problems using multiplication operations on one glass and two glasses. It should only use the sum operation on one glass. In problem 2, S10 has problems using multiplication operations on two racks. Should have used surgery. In problem 3. S10 has a problem using the multiplication operation needed to use the operation twice. In question number 4, S10 has a problem also using the multiplication operation. It should use a count operation and wait. In

problem number 5, S10 has the problem of using multiplication operations three times. It should use surgery at time one and operation at time two.

Subject 12

In questions no. 1 to no. 5, S 12 uses multiplication count operations, while this arithmetic operation is not used to solve problems from no1 to number 5. S12 writing is small and difficult to read. Subject 13 and subject 14

In questions no. 1 to 5, S13 and S14 do not include any arithmetic operations. There are only fraction answers in the answer.

Subject 15

Subject 15 did not include any arithmetic operations and the writing S15 was irregular and could not be written

Subject 1

In question no. 5, S1 does not transform numbers 1 and 1/3 into the subtraction calculation operation. Next is the S1 cutout sheet that made this mistake.



Figure 8. Error S1 does not transform the fraction number

Subject 2

In problem number 5, like S1, S2 also does not transform the numbers 1 and 1/3 into the subtraction calculation operation. This error causes incomplete answer completion.

Subject 4

In problem number 2, S3 incorrectly transforms 1/8 into 8/1. This can cause errors in subsequent calculations.

Subject 8

In problem number 3, S8 does not transform 1/5 into arithmetic operations. This causes subsequent calculations to be incomplete. In problem 5, S8 does not transform number 1 into a count operation.

Subject 9

In question no 5. S9 does not transform 1/3 into arithmetic operations

Subject 10

In question no. 1, S9 misplaced 11/2 and 2 1/4 into arithmetic operations

Subject 12, subject 13, subject 14 and subject 15

In questions no. 1 to no. 5, S12 does not transform known fraction numbers into the formula correctly and most do not transform known numbers. In questions 1 through 5, S13 also does not transform all fractions into the formula correctly. Likewise for S14 and S15.

d. Error counting process

1) Does not equate to the denominator

Subject 4, subject 6, subject 7 and subject 8

Subject 4 in questions no. 2, S6 do not equate the denominator in questions 1 and 2, S 7 in question number 1 and questions number 5, S 8 in questions number 1, 2 and no 5, do not equate the

denominator immediately subtract the numerator. The following clippings of the S4 answer sheet do not equate to the denominator.



Figure 9. Error S4 does not equate the denominator

2) Incomplete writing process

Subject 11

In problem number 5, S11 can already equate the denominator but cannot determine the numerator so S11 cannot complete the answer. The following clipping of the S11 answer sheet is incomplete in writing the process



Figure 10. Error S11 is incomplete in write down the process

Subject 3

In question no. 4, don't write the subtraction process in step one and stage two because the arithmetic operations used are wrong

Subject 4

In problem 4 do not write the addition process.

Subjects 5, 6, and 7

In questions no. 1 and 4, S 5 does not know that to equate the denominator must determine the KPK from the numbers in the denominator of the fraction, so that S4 has difficulty in equating the denominator. S6 in questions 1 and 2 also do not equate the denominator. S7 on question no 4 does not change the numerator of the previous fraction.

3) Miscalculate

Subject 4,5,9,10,11

S4 in question no2 is wrong in counting, i.e. changes 1/8 to 8/1. The following clipping of the S4 answer sheet made a mistake.



Figure 11. S4 error in counting

S5 in question no. 4 is wrong in equating the denominator and the denominator is added and subtracted according to the arithmetic operations used. S8 in questions 2 and 4 mistakenly

calculated which should be reduced to division., S10 cannot equate the denominator in question no. 1.

4) Don't change the numerator

Subject 8,

In question no. 2 S8 does not change the numerator and the numerator results are the subtraction of the two numbers in the numerator. Next is the cutout of the S8 answer sheet that made a mistake



Figure 12. S8 error does not change the numerator

5) Do not do the counting process

Subject 6

In questions no. 1 to questions no 5. S 6 does not include the calculation process, there is only the final result. Following is the clipping of the answer sheet S6 which made a mistake.



Figure 13. Error S6 does not do the counting process

6) Incorrect use of methods

Subjects 2,4,5,6,9 and 13

In question no. 5 S13 it is wrong to use the method, this is a mistake at the transformation stage even though the calculation is done correctly according to the operation of the calculation. In question no. 5 almost all subjects (S2,3,4,5,6,9 and 13) using the wrong method which is an error caused at the transformation stage, the subject is fooled from the question in the problem. Almost all subjects do with the new addition process followed by subtraction through one stage, whereas it should be through two stages namely stage one using subtraction operations and stage two using subtraction operations as well. Next, cut out answer sheet Following cutout answer sheet S13 in question number 5 is wrong in using the method



Figure 14. S13 errors in using methods

#### e. Error writing answer

Based on the recapitulation of student errors, the highest number of errors made by students lies in writing answers. This happens because most students have made mistakes in the previous stage and few students can solve them until the answer writing stage. Here are the types of errors that researchers found in this study.

1) Do not write down conclusions

Subjects 1 and 3.

Subject 1 has done the calculation phase on problem number 2 correctly but did not provide a conclusion. subject 3 also performed calculations with complete processes and correct in questions number 1 to 4, but do not write the conclusions as well. Following the cutout answer sheet S3 who made a mistake.



Figure 15. S3 error in writing answers

Table 4.1 Recapitulation of Students'	Errors in Working on Fractional	Material Story Questions
1	0	

Error type	Number				Sum	
	1	2	3	4	5	
Read	3	3	3	3	3	15
Understand	12	12	13	13	13	63
Transformation	4	9	9	11	13	46
Counting process	11	10	13	14	14	62
writing answer	14	14	15	15	14	72
Total	44	48	53	56	57	258

Based on the data in table 4.1 it can be seen the number of errors made by students from each problem and the type of error. In question number 1 the most mistakes made by students were error writing answers as many as 14 times. In question number 2 the most mistakes students make are the

mistakes in writing answers that are 14 times. In question number 3 the most mistakes students make are the mistakes in writing answers 15 times. In question number 4 the most mistakes students make are mistakes in writing answers, which is 15 times. Whereas in question number 5, the most mistakes made by students were calculation errors and writing errors of answers as many as 14 times each.

If seen from the number of errors in each question, then the most mistakes made by students are in problem number 5 and the least mistakes made by students are in problem number 1. This is in accordance with the difficulty level of the problem, where question number 1 is an easy problem and question number 5 is a difficult problem. When viewed from the type of error, the most mistakes made by students are errors in writing answers and errors that are rarely made by students are reading errors.

2. Findings of the results of student interviews regarding the causes of student errors in working on fraction story problems

In this study, interviews were conducted on fifth-grade students of Elementary School in Serang City. The purpose of this interview is to obtain information about the causes of student errors in working on fraction story problems. Interviews were conducted by six students with researchers to 15 students after the test was completed. Each student interviewed 2 students and the researcher interviewed 3 students. The following is a description of the results of student interviews.

a. Subject 1

Problem number 1

S1 made mistakes on questions number 1 to number 5 by not writing the unknown and asked, based on the results of the interview it was known because they were accustomed to working on story problems and went along with friends not to write things known and asked.

Problem number 5

SI made a mistake in transforming the calculated operation and the fraction. From the results of the S1 interview, he stated that he did not understand the purpose of the contents of the problem so that it was wrong to move the fraction to the answer completion.

b. Subject 2

Problem number 2

The results of the interview obtained information that students do not write down known and asked. Students are wrong in writing arithmetic operations because words given according to S2 are the same as being divided so that the reduction marks are made by students into divisions. The calculation process done by Subject 2 is correct, but because the marks used are wrong even though the counting process is correct, the results of the answer of subject 2 in number 2 are still wrong. Subject 2 states that he is accustomed to working on story problems by not writing things known and asked so answers number 1 through number 5 do not write things known and asked.

c. Subject 3

Problem number 1 through number 5

Subject 3 does not write the things that are known and asked because they are accustomed to doing it when answering or solving a story problem. Subject 3 states that he understood the meaning of question number 1 through number 4, but for question number 5 because he read it over and over and did not really understand. In question number 4 subject 3 states that it is still difficult in simplifying fractions so that the final results of the calculation process are not completed. Subject 3 states that he had no difficulty when equating the denominator by finding the KPK from the numbers on the denominator. In problem number 5, subject 3 found it difficult to understand so that

he did not transform the fraction and its calculation operations correctly. Subject 3 states that he is accustomed to not concluding answers when answering story questions. Subject 3 understands the purpose of question number 1 through number 4, this corresponds to the answer of subject 3 which is correct from answer number 1 to number 3, and in problem number 4 the calculation process is correct, only the final result is still wrong.

#### d. Subject 4

Problem number 2, 4, and 5

Subject 4 also stated that he was accustomed to not writing things that were known and asked, so he did not write them on question number 1 to question number 5. Based on interview subject 4 for problem number 2. Subject 4 stated that it could not equalize the denominator, he was less understand that the KPK is used to hide denominators. Subject 4 also stated forget not to include conclusions in the answers to questions number 1 to problem number 5.

#### e. Subject 5

Questions number 1 through problem number 5, subject 15 there is not a single correct answer. Based on the interview results, it turns out that the students are inclusive students. This can be seen from the answers of students who could not answer the questions given and the writings of students who could not be understood as well as random. The obstacle in learning fractions in elementary school found in this study is the difficulty in understanding fractions similar to the research that has been done by Smith, Solomon, & Carey, (2005). Vamvakoussi, & Vosniadou, (2010), Yetkin, E. (2003), Ross, & Bruce, (2009), Siegler, Fazio, Bailey, & Zhou, (2013).

## **Teacher Interview Results to Find Out Solutions to Overcome Student Mistakes**

The results of interviews conducted with teachers: a) When learning fraction topics the teacher uses the demonstrate method and lectures and uses concrete media such as wafer pieces to demonstrate the addition of fractions with the same denominator. When teaching fraction story problems the teacher gives a few examples of questions and solves the problem through question and answer, then groups students and provides worksheets that contain problem-solving questions. Through this grouping the teacher expects students to work together and help each other to solve the problem. According to the teacher, the obstacles and difficulties experienced in teaching fraction story problems are students often have difficulty in understanding story problems so they often ask questions and require more time to work on them. In addition, researchers also asked mistakes that were often made by students in working on fraction story problems. The teacher replies that the errors that students often make are mistakes in determining arithmetic operations. This happens because students have difficulty in the stage of understanding so often wrong in using arithmetic operations, for example, which should be reduced to division or vice versa. The final question asked is what steps can the teacher take to minimize student errors in working on story problems. According to the teacher, students are given questions about routine problems instead of solving problems with quite a lot of practice and practice solving story problems with varied problems. The difficulties in learning fractions in elementary school found in this study are the same as the research conducted by Lortie-Forgues, Tian, & Siegler, R. S. (2015), Hecht, Vagi, & Torgesen, (2007), Torbeyns, Schneider,, Xin, & Siegler, (2015), Steffe, & Parr (1968), Radu, McCarthy, &

Kao, (2016) and Leu, (1999).

# 4. Conclusion

In general, students' mistakes in working on story problems are mistakes in reading story problems, understanding questions, transformation, calculation process and writing error answers. The highest mistake in understanding the problem. The ability of students to connect mathematical concepts with story sentences requires habituation from the teacher. Fractional learning if delivered with lectures, students will find it difficult to understand fractions. Fractional learning will be more easily understood with concrete media and the time in doing it must be adjusted to the problems the students are working on.

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