

**ECONOMIC ANALYSIS OF ENTERPRISE THROUGH PRODUCTION FUNCTIONS  
WITH COMPLEX VARIABLES**

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**Abstract**

The paper looks at the basis of modern theory of functions with complex variables and explores possibilities for their usage in enterprise analysis. Elaboration and analysis of economic models with complex variables is described with the purpose of revealing characteristics, which can be used in analysis and planning of enterprise activities.

**Abstracto**

El papel mira la base de la teoría moderna de funciones con variables complejas y explora posibilidades para su análisis en enterprise. La elaboración y el análisis de modelos económicos con variables complejas se describen con el objetivo de revelar características, que se pueden usar en análisis y planificación de actividades de la empresa.

## Introduction

Analysis, planning and forecasting of production processes are essential in decision making at any enterprise. One way to solve such problems is to use production functions. In the case of a well known Cobb - Douglas production function coefficients are determined based on the method of least squares on series of data, and then based on the value of factor  $\alpha$  one can make conclusions on the nature of a production process, give recommendations about its improvement and forecast what quantity of products to produce with the given technologies. In the case of production functions with complex arguments and complex variables, the tasks of an analyst are about:

- analysis- finding coefficients on each observation and assessing their dynamics. Forecasting - finding coefficients from the whole row of data based on the method of least squares and forecasting the outcome of keeping production process unchanged and changing factors production [1].
- planning - formulating inverse production functions, which help calculating, the amounts and values of input in order to achieve a certain output level [2].

## Main part

In this paper we will look through and compare four production functions on the data from "Megasfera" (the Republic Uzbekistan, Urgench) Ltd. Enterprise, namely:

1. Cobb – Douglas:

$$Q = AK^\alpha L^{1-\alpha};$$

2. Complex argument:

$$Q = a(K + iL)^b$$

## 3. Complex variables with real coefficients:

$$Q + iC = a(K + iL)^b$$

## 4. Complex variables with complex coefficients:

$$Q + iC = (a_0 + a_1i)(K + iL)^{b_0 + ib_1}$$

Here; K is capital expenditures, L is labor expenditures, Q is quantity the production, G is profit, C is costs of production,  $A, \alpha, a, b, a_0, a_1, b_0, b_1$  - coefficients of corresponding functions.

We will begin with Cobb - Douglas production function (1).

Based on the real data from the enterprise from first quarter of 2012 till second quarter of 2013 the Cobb-Douglas production function took the following shape:

$$Q = 1,407 K^{0,457} L^{0,543} \quad (5)$$

The model describes the dynamics of production process very well; approximated average is equal to 14,8%. Moreover, coefficient  $\alpha$  equals 0,457, which drives us to a conclusion that the production process is labour-intensive. We have looked through several options of further development of the enterprise (increase and decrease of capital and labor inputs). According to the results from Cobb-Douglas production function overall conclusion for "Megasfera" Ltd. is that enterprise has to enlarge investment in the production assets. It is also possible to recommend to enterprise to enlarge the number of workers - so enterprise "Megasfera", will be able to increase its output level, decreasing the number of employed workers is not recommended, since this will provide reduction of quantity as a result of labour-intensive process.

Now let's consider the production function with complex argument (2). In this function of interest is the indicator  $b$ . We will find its value for each observation on the data of "Megasfera", (quarterly

data from first quarter of 2012 till second quarter of 2013). Values of coefficients are presented in the following table:

Table 1. Values of b-coefficient

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14
b	8,00	8,23	8,19	8,67	8,89	9,23	9,12	9,45	9,49	9,86	10,15	10,50	11,23	11,43

The growing value of coefficient b means that production in enterprise is becoming more capital intensive each quarter, which leads to increase in labor productivity

The production function with complex argument with real coefficients for the last observation will take the form of:

$$Q = 0,02(K + iL)^{10,45} \quad (6)$$

Based on the obtained results, the enterprise is recommended to continue investing in fixed assets for improving production process.

For function (3) we have calculated value of b, value of the critical point of  $b_1$  and value of coefficient efficiency S, which calculated according to the following equation:

$$S = 1 - \frac{b - b_1}{b + b_1} \quad (7)$$

All these coefficients, as well as the values of production factors and their relative values are presented in Table 2 below.

Table 2. Coefficient values for the estimated Cobb-Douglas production function

Quarter	K	L	a	b	b <sub>1</sub>	S
1 <sup>st</sup> 2012	1,00	1,00	0,502	1,199	0,52	42,10%
2 <sup>nd</sup> 2012	1,07	0,95	0,96	2,12	0,55	41,30%
3 <sup>rd</sup> 2012	1,09	1,03	0,79	2,15	0,61	43,95%
4 <sup>th</sup> 2012	1,15	1,04	0,69	2,16	0,73	50,98%
1 <sup>st</sup> 2013	1,16	1,05	0,50	2,16	0,76	51,15%
2 <sup>nd</sup> 2013	1,19	0,95	0,52	2,40	0,84	53,13%
3 <sup>rd</sup> 2013	1,21	1,06	0,55	2,20	0,86	55,07%
4 <sup>th</sup> 2013	1,33	1,08	0,59	2,34	0,98	59,20%
1 <sup>st</sup> 2014	1,35	1,09	0,50	2,50	1,05	61,35%
2 <sup>nd</sup> 2014	1,42	1,02	0,47	2,60	1,10	64,34%
3 <sup>rd</sup> 2014	1,49	1,03	0,39	2,63	1,34	67,45%
4 <sup>th</sup> 2014	1,52	1,07	0,33	2,70	1,48	70,54%

The most important for us is the value of coefficient of efficiency S. It shows that the efficiency of the enterprise is increasing each quarter. It is noticeable that the value of the fixed assets is also increasing, corresponding to efficiency, from quarter to quarter, whereas the number of the labour L does not

change. This shows that that investments in fixed assets increase the efficiency of enterprise activities.

Our last observed production function with complex variables takes the following form:

$$Q + iC = 0,29(K + iL)^{2,32} \quad (8)$$

Having considered the same example with options for further development of the enterprise, we conclude that the enterprise can maximize income, as well as profit by decreasing labour and increasing the value of fixed assets. The lowest values of production costs can be obtained through reducing both the number of employed labour, as well as the value of fixed assets. According to the results, the enterprise needs to reconsider its employment policy since given the current set of production factors the enterprise can perform more efficiently. For understanding how the enterprise can become more efficient and what is required for this, let's develop an inverse production function to the function (8).

To begin with, we shall consider the values of production costs and profits from the last observation under the condition that the enterprise works up to its full efficiency, that is to say,  $b=b_1=1,48$ . For this we will take values of  $K$ ,  $L$ ,  $a$  and  $b_1$  for 4<sup>th</sup> quarter of 2014 and calculate values of  $G$  and  $S$ . We will get the following positive values:

$$G=0,550 \quad S=0,642 \quad (9)$$

In absolute values this will be:  $G=15,568$  mln.UZS(7076 USD),  $C=16,94$  mln.UZS(7700 USD).

Considering all this, we will get following absolute values of capital and labour:

$$K= 102,500 \text{mln.UZS}(46,590 \text{ thousand USD}). \quad (10)$$

For comparison it is worth mentioning that in 4<sup>th</sup> quarter of 2014 enterprise's value of the fixed assets was 117,500 mln.UZS(53,409 thousand USD) but the number of labour force was 140 people.

The result of (10) however does not indicate that the enterprise must urgently decrease two-fold the

number of employed workers and sell the technology. The result indicates that the enterprise has to optimize labour productivity by redistributing places occupied between production and non-production departments. It is hard to provide with more detailed recommendations based on the calculations and obtained results, which will require deeper analysis of the enterprise's activities.

Let's construct the last production function with complex variables (4). Calculations of the coefficients of function (4) by means of the least squares method, gives us the following production function:

$$Q + iC = (0,26 + i1,56)(K + iL)^{0,43+i0,04} \quad (11)$$

The average mistake to approximations for function (11) has formed 14,87%.

Calculation of the same as 9 development options for production function (11) gives us the same information, as with the production function (8).

## Conclusion

Based on the overall results of the analysis we can derive the following conclusions about the enterprise "Megasfera":

1. Increased investment in fixed assets from quarter to quarter helps to raise labor productivity, and labor becomes more efficient.
2. In established production process enterprise can work more efficiently. That is why it is necessary to improve labour policy of enterprise and reduce the number of labour in production (or optimize labour force), as well as continue investing in fixed assets.

With regards to the production functions, we can conclude that functions with complex argument and production functions with complex variables provide more information about the details of the

current production processes at a given enterprise, compared to the Cobb - Douglas production function with real variables.

The results received from Cobb - Douglas production function do not lead to the conclusion or recommendation to reduce the number of workers in the production process, on the contrary, the function shows that the increase in labour leads to the increase in production volumes. Other functions described in the study, urgently recommend enterprise "Megasfera" to think about the reduction of labour force, or optimization of the structure of labour in enterprise, thus leading to balanced management decisions.

## References

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